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STUDY OF THE OCCURRENCE OF RAIN IN THE TUNIS AREA IN A MONO-FRACTAL FRAMEWORK

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ABSTRACT
Less study concern the scale variability of the rainfall field in North Africa (Mediterranean climate), where the climate differ significantly from the Sahelian Africa (monsoon climate). This paper undertakes a study of the occurrence of rain in the region of Tunis in a mono-fractal framework. The box-counting method is applied to four series of observations of a continuous period of two and a half years, based on a minimum resolution of 5 min and belonging to the semi-arid bioclimatic stage. These series are characterized by strong intermittency. Using the sensor detection threshold, two self-similar structures were detected: micro-scale (5min- 2 days) with fractal dimension 0.44 and a synoptic-scale (one week – eight months) with fractal dimension of 0.9. This last value is probably overestimated by the presence of the saturation of the available space by rain (fractal dimension equal to one) for period longer than eight month. Due to the length of the dry period observed, the length of this saturation period differs from other studies performed in other area. Between the two self-similar structures a transitional regime corresponding to a meso-scale (2 days- one week) could be distinguished. The increase of the threshold would allow to 'filter' the frontal structure so as to keep only the convective structures, a sub micro-scale structure (5mn – 1h 20) has been detected with 0.3 mm/5mn intensity threshold. These results may reflect the influence of two distinctive types of convective showers and original front controlling these series.

Keywords: Semi-arid Mediterranean climate; fractal dimension; rainfall time series; scale-invariance; on-off intermittency; mono-fractal ; box counting.

1 INTRODUCTION

The rainfall is a process which generally shows a high spatial and temporal variability. This variability is controlled mainly by intermittence (occurrence – non-occurrence). There are no studies dealing with the rainfall occurrence in the semi arid Mediterranean climate. This region is characterized by two seasons: a rainy one and a dry one. The average annual rainfall is between 600 and 900 mm. The rain support can be considered as a fractal object (Lovejoy, 1981; 1982) and is characterized as being a non-integer 'fractal dimension' (Mandelbrot, 1975). This dimension is interpreted as the degree of irregularity according to which the support is distributed. In using the method of box counting, the present paper deals with the rainfall occurrence by studying the fractal dimension of the support of precipitations and its properties of scale invariance. Numerous papers have put into evidence that the process of rainfall is characterized by a hierarchy of fractals leading to a multi-fractal or multi-scales process (Biaou, 2004). Schertzer & Lovejoy (1987) have developed a universal model (multi-fractal) as well as different methods of identification of its parameters. Recent works (Montéra., 2009 ; Verrier et al., 2010; 2011) have noticed a bias on the estimation of parameters of the multi-fractal model on the intermittent series as well as the appearance of breaks on the scaling separating various regimes. The study of the fractal dimension of the support and the regions of scale invariance is then considered as an indispensable preamble to the multifractal analysis. The following Section 2 presents the available rainfall data in the region of Tunis. Section 3 present the analysis of the rain support obtained on 5-minutes series. In the last section some concluding remarks are presented.

2 BRIEF DESCRIPTION OF GRAND TUNIS AVAILABLE RAINFALL TIME SERIES

This study examines the pluviographic and pluviometric informations in Tunis area. Four automatic rain gauges that cover an area of 265 km² are analysed.The data used for this study come from the database provided by the General Directorate of Water Resources (Direction Générale des Ressources en Eau (DGRE)) of the Ministry of Agriculture. Main caractéristics of the time series used in this study are given in

Table I and Figure 1. These data come from the recording of automatic rain gauges recording by tipping buckets rainy gauges. The series are sampled by time step of 5 minutes. The accuracy of recorded observations is 0.1 mm. The original series have a length varying between one and two years and a half, the available duration corresponds to the maximal length squared two (for the necessity of calculation related to the method of box counting used later on) that we can obtain. We call rainfall event, every period of rainfall located between two non- rainy periods.

Table I – The available data set

Stations	Tunis-Manoubia	Mornag	Soukra	Sidi Thabet
Longitude	8Gr, 7060	8Gr, 8500	8Gr, 3770	8Gr, 5580
Latitude	40Gr, 8711	40Gr, 6600	40Gr, 9540	41Gr, 0040
Altitude (m)	66	35	15	20
Start date	31-Dec 2007	31-Dec 2007	31-Dec 2007	31-Aug-2007
End date	28-Jun-2010	28-Jun-2010	30-Mar-2009	26-Feb-2010
Length of series(minutes)	$5*2^{18}$	$5*2^{18}$	$5*2^{17}$	$5*2^{18}$
% of zero	99.17	98.99	99.33	99.04
Number of events in the series	1092	1298	480	1508
Cumulative duration of events (days)	7.5	9.1	3.0	8.8
Average Duration of events (minutes)	9.87	10.12	9.13	8.36
Maximum duration of events (minutes)	195	235	255	220
Average length of dry periods (hours)	19	16	14	20
Max duration of dry period (days)	74	88	124	117

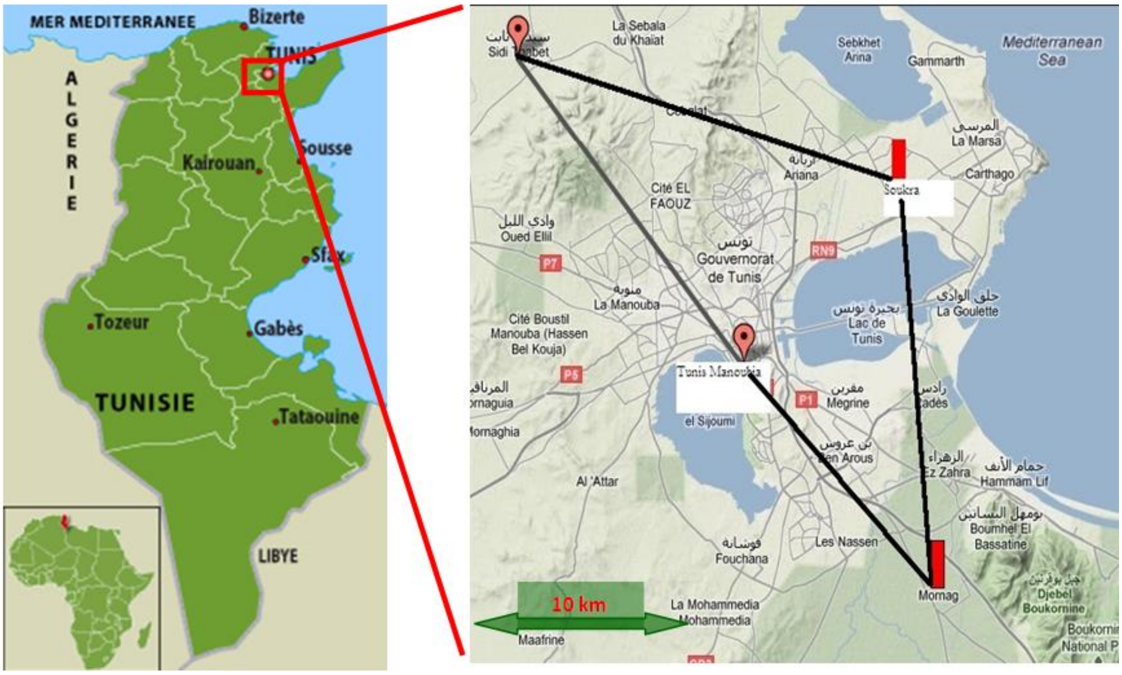

Figure 1 – Localization of the stations.

We notice a good coherence of characteristics on the set of stations. On the whole, the weakness of durations cumulated in relation to the duration of sequences provides an idea of the scarcity of rainfall waters. In fact, most of the time, it is dry with a percentage of zeros higher or equal to 99% for the four stations. The rainfall is concentrated mainly between the months of September and May in a limited number of days. They are almost absent during the summer months (June, July and August). The statistical distribution of event duration is extremely asymetrical. The first quartile and the median are superposed and equal to 5 minutes. More than half of the events correspond therefore only to one isolated observed non null data. The third quartile is equal to 10 minutes for both stations of Tunis-Manoubia and Mornag (situated in the south of the studied zone). However, it is comprised between 5 and 10 minutes for the stations of Sidi Thabet and Soukra (situated in the north of the studied zone).

3 STRUCTURE OF RAIN SUPPORT WITH A BOX COUNTING APPROACH

We try to characterize the rainfall structure by the fractal dimension of its occurrence. The fractal dimension of rain occurrence is calculated by using the method of box counting. The total observation time T is divided into n contiguous intervals of length *a* taken as successive powers of two. Thus, T= na. The total number of occupied intervals N(a), that is, those in which at least one raindrop has been observed, is then counted. If the data form a one-dimension fractal, then we must have

$$N(a) \propto a^{-D_f} \qquad (1)$$

where Df is defined as the fractal dimension of the process. When plotting N(a) as a function of log(a), a straight line with slope - Df is obtained. Of course, when a→ T, N(a) → 1, and, asymptotically, one should have $\lim_{a \to T} N(a) = n = T/a$

Two box counting methods are suggested by Olsson et al. (1992). The first method consists in taking a null threshold of rain detection (ie we consider the whole rain structure). In the second method, the rain occurrence is analyzed at various intensity thresholds.

3.1 With the sensor detection threshold

The fractal dimension of rain occurrence is firstly evaluated by adopting the zero thresholds when applying of the box counting. The results of 18 counting performed for each of the four stations for links of time series are considered according to boxes, the length of which are the terms of a geometric progression of term 1 and of reason 2 multiplied by 5 minutes, are presented in Figure 2.

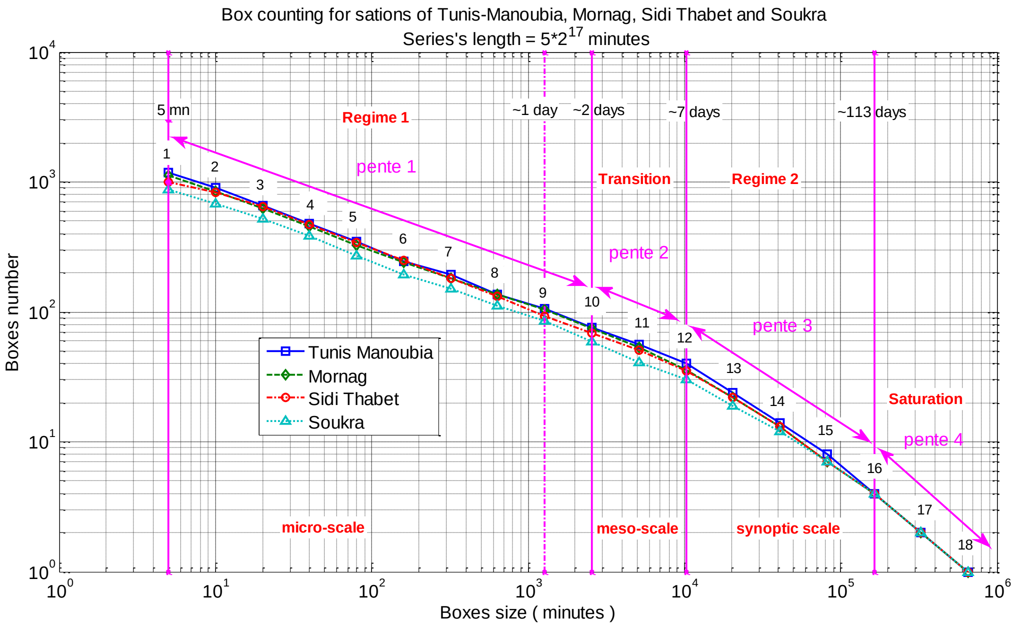

Figure 2 – Box counting from rainfall series of the stations of Tunis-Manoubia, Mornag, Sidi Thabet and Soukra

The breaks on the graphs observed for the four stations at points numbered 10, 12 and 16 which correspond to four gradient changes are identified around boxes of size 2 days, 7 days and 113 days. The first regime between 5 mn and 1 day is characteristic of the micro-scale. It characterizes the internal structure of rainfall events. The transition [1 day – 8 days] corresponds to the meso-scale rainfall structures whose spatial extension is from 20 to 200 km, characterizing the frontal systems. The second regime between 7 days to 8 months corresponds to the synoptic scales linked to the general circulation.The last alignment of points 16 to 18, which corresponds to the saturation of the available space by rain and a fractal dimension equal to one, reflects a behavior underlined by all the authors. Only the length of the saturation period differs from one author to another in the literature review. It is in fact directly related to the length of the dry period.

Considering the works of Olsson (1993) starting from the rainfall data collected at Lund (Sweden), Hubert and Carbonel (1989) obtained in Ouagadougou (Burkina Faso) as well as those of Lavergnat & Golé (1998) collected at Palaiseau (France) and those of Schmitt et al. (1998) obtained at Uccle (Belgium) or those of Veneziano & Iacobellis (2002) collected at Florence (Italy), we notice that the scale break from which we get a saturation is observed around few days witch attests the very important length of the saturation regime confirming the relative homogeneity of the rain support throughout the year. In other word the length of the saturation period is very particular to the considered area. Therefore, contrary to studies performed in less contrasted climates (we can observed at least 10 minutes of rain for any periods of 3.5 days in Belgium or at least 1 minutes of rain for any periods of 7 days in Sweden), precipitation are not systematically observed over periods of few days, we observe, in our case, at least 5 minutes of rain only for periods higher or equal to 113 days. Due to the presence of a 4 months dry period in Mediterranean semi-arid area, it is not surprising to observe a particular behavior concerning the length of the saturation period.

Table II – fractal dimension of the four stations

		D_f	R^2
Regime 1 Alignment of points 1 to 10 5mn to 2 days	Tunis Manoubia	0,44	0.999
	Mornag	0.43	0.999
	Sidi Thabet	0.44	0.997
	Soukra	0.43	0.999
Transition Alignment of points 10 to 12 2 days to 7 days	Tunis Manoubia	0.46	-
	Mornag	0.53	-
	Sidi Thabet	0.49	-
	Soukra	0.49	-
Regime 2 Alignment des points 12 to 16 7 days à 113 days	Tunis Manoubia	0.82	0.996
	Mornag	0.80	0.998
	Sidi Thabet	0.79	0.997
	Soukra	0.73	0.998

The three other groups of aligned points are analysed and corresponding characteristics, the fractal dimensions of the occurrence of rainfall Df (Equation 1) as well as the coefficients of determination R2 are given in Table II. We observe a behavior very similar for the four available times series.

For the second regime, we obtain a fractal dimension between 0.79 and 0.82. The series relating to the station of Soukra is therefore too short to offer a reliable description of this regime. We notice that the estimation of this dimension is very sensitive to the length of used series. The analysis of much longer daily series, that will be performed in further study, is much better suited to the study of this regime characteristic of synoptic scales. However, the coefficients of determination (R2) higher than 0.995, attest of the quality of obtained alignments.

In the case of the transition zone (alignment of points 10 to 12) the scale ratios are too weak since the fractal dimension is estimated with only 3 points and it is thus hard to judge the quality of the alignment that is why R2 has not been mentioned for this case. The obtained fractal dimensions Df are between 0.46 and 0.53. This intermediate alignment can be considered as a regime of transition between first regime which characterizes the thin structure of the support and the second regime which characterizes a big scale.

Indeed, the first regime of scaling has a fractal dimension between 0.43 and 0.44. The existence of several self-similar structures for the finer structure of rainfall is mentioned by many authors with a fractal dimension, however, a little different from the one obtained here since not only Schmitt et al. (1998) and de Lima & de Lima (2009) with a resolution of 10 minutes, but also Verrier et al. (2011) with a resolution of 15 seconds have obtained a fractal dimension of 0.55. On their side Veneziano & Iacobellis (2002) found a fractal dimension of 0.5 for time increments ranging from 20 min to 3.5 days for a time series resolution of 20 min. Hubert & Carbonnel (1989) have obtained a dimension equal to 0.22 for a scale ranging from one hour to 32 hours using a time series of hourly resolution of only 170 days long. Lavergnat & Golé (1998) do not observe this regime, but they use only four months of observations. The results of Olsson et al. (1992) which identify a break in scale at 45 min are not coherent with ours. These differences can be attributed either to the differences of rain structure (previous studies do not concern the semi arid Mediterranean climate studied here), or to differences of rainfall threshold detection of different used devices. In fact, we have to underline that the zero threshold , is an arbitrary threshold related to sensor sensibility and to its time resolution.

3.2 Application of different thresholds of rain intensity

We are going to consider the evolution of scaling regimes and of the fractal dimension by changing the threshold of rain detection. In fact, the use of data without a threshold integrates all rainfall structure so the convective peaks are not separated from frontal rain. By increasing the threshold, the rainfall structure changes and the scaling regimes could be modified. Nevertheless, by using increasingly high thresholds, we erase a portion of rainfall information (frontal part).

The structure of rain support is not modified by a weak increase of the detection threshold. The structures and gradients obtained are identical for the thresholds 0 ; 0.1 and 0.2 mm/5mn. From a threshold ≥ 0.3mm/5mn (3.6 mm/h) the structure of the rainfall field change and we observe a further break corresponding to 1h 20 min. Figure 3 shows the results of box counting for the Mornag station for thresholds 0; 0.4 and 1 mm. The fractal dimension of the regime corresponding to the box sizes from 5 min to 1 h 20 min (0.37) is higher to that of the regime ranging from 1 h 20 min à 2 days (0.24), but they remain both lower than the relative fractal dimension with a zero threshold (0.43). Likewise for the other observed regimes, the fractal dimension decreases. The break observed at 1h 20 persists for larger thresholds with a decrease in the fractal dimension. The curve patterns thus obtained is coherent with the previous works of Olsson (1992, 1993) in the case of series at 1 min. resolution. However, this author observes a break of 45 minutes with values of the fractal dimension much greater.

This behavior is related to the Mediterranean regime of the region of Tunis. The intense precipitations are due to isolated convective cloud structures of big dimensions, having a lifetime of a few hours during which the events occur in a collective way. The increase of the threshold would allow to 'filter' the frontal structure so as to keep only the convective structures. In fact, while considering higher thresholds of rainfall intensity the structure of convective events appears progressively. Convective structures would present properties of scale invariance between 5 min and 1hrs 20mn. The low value of the fractal dimension of this regime can be interpreted as a high degree of irregularity of distribution of the support of these structures.

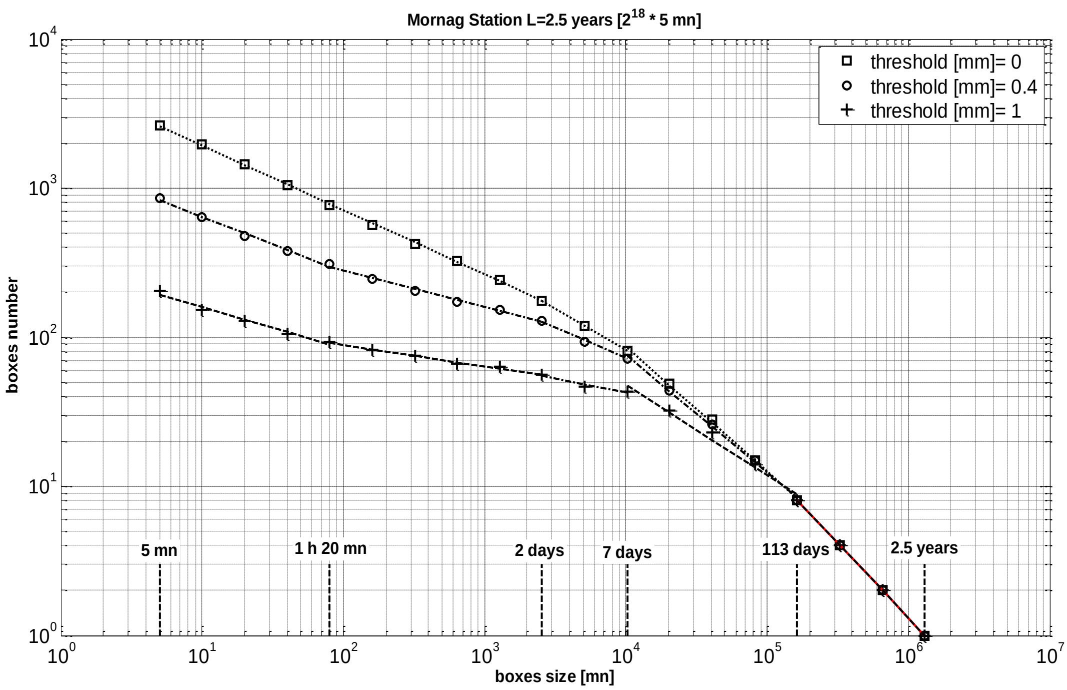

Figure 3 – Box counting derived from rainfall series of 5-minute time step, of size of 2.5 years of the station of Mornag for rain intensities: 0; 0,4 and 1 mm per 5 min. Straight lines are linear regression adjustments

4 CONCLUSIONS

Analysis of the box-counting plots of 4 series of observations with sampling time of 5 minutes has clearly shows that rainfall present properties of scale in a mono-fractal framework. Four time intervals that present scaling properties are identified. A first one, characteristic of the micro-scale, which characterizes the intern structure of rainfall events and which concerns the time scale between 5 min and 1 day is shown. A transition [1 day – 8 days] that corresponds to the meso-scale rainfall structures. A second regime between 7 days to 8 months that corresponds to the synoptic scales. The last one corresponds to saturation of the available space by rain.

We get for the first rainy regime a fractal dimension of 0.44. This micro-scale behavior, which characterizes the internal structure of rainfall events, is mentioned by several authors with a fractal dimension, however slightly different, around 0.55 for oceanic and subtropical instead of 0.44. The analysis performed with different threshold on the 5 minutes times series have shown a break near 1h20 that can be interpreted as a different scaling regime characteristic of convective situation. The convective part of the process appears progressively when the threshold of rainfall intensity increases. The 5 minutes resolution of available data, does not allow us to analyze per event in order to distinguish the structure of convective events from those of frontal system. Higher resolution data are necessary to refine the analysis of the micro structure of rain.

The feature that characterizes the study area concerns the saturation period. We obtain a beginning of the saturation period after 8 month when other authors observed saturation after a few days. The saturation of the available space by rain occurrence is a trivial consequence of significant length of the dry period. This result highlights a peculiarity of rainfall of the semi-arid Mediterranean climate, characterized by prolonged dry periods and rare isolated event.

So that the rainfall process cannot be wholly described by a single fractal dimension since the process is characterized by a hierarchy of dimensions. So, as continuation of this work, we will study the process of rain in Tunis area in multifractal context for surrounding and getting a clear idea on the hydrology of the region.

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